Emergency Vehicle Facility Location

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Master Dissertation

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À minha família, sem ela nada seria possível.

Abstract

Decreasing the elapsed time to respond to an emergency is the primary goal of any emergency service around the world, since every second is vital to minimize human losses. This way, there are formulated several problems to pursue this goal as, for example, shortest path, fleet sizing and location and/or relocation of stations or vehicles. In this research, we focus on fire fighting services, more specifically in the definition of the best resource distribution, i.e. study of the localization problem, one of the main problems encountered by planners.
Furthermore, empirical studies point out that the probability of occurrence of emergencies in urban regions and the level of life risk associated with are strongly related with the concentration of the population. Hence, it is important to consider the different dynamic changes during the day when locating the fleet. Most existing approaches in literature fix travel time between locations in the studied city and, consequently, consider fleet locations constant overtime. The proposed in this dissertation is to find flexible locations for fire fighting vehicles during specific periods of the day taking into account the speed fluctuation in the streets and the expected probability of occurrence of emergencies, estimated based on the population migration. Thus, we formulate an integer linear programming model in order to temporary locate and/or relocate fire fighting vehicles to maximize the emergency coverage in the required time limit.
Based on two different problems of municipalities of the city of Santiago, Chile, we show that this approach leads to improvement in the coverage of the studied regions from overall coverage level of around 63% to around 97% (arrivals at the site in less than 7 minutes after reception of the alert). There is also shown the best configuration that would guarantee total coverage in the municipality of San Bernardo.
Additionally, the model is adaptable to the system flexibility on positioning vehicles outside fire stations, according to the specific legal and operational limitations of each case.
Although it was possible to model a real problem, the length of the dissertation was not able to gather all the data needed to calculate the operational costs associated with the vehicles positioning of the proposed solution.

Resumo

Diminuir o intervalo de tempo decorrido entre a chegada de uma chamada de emergência e o seu atendimento no local é o objetivo central de qualquer serviço de emergência no mundo, já que este é crucial para diminuir as perdas em vidas humanas. Desta forma, são formulados vários problemas para perseguir este objetivo como, por exemplo, caminho mais curto, dimensionamento de frota e localização e relocalização de estações ou veículos. Esta investigação foca-se nos serviços de combate de incêndios, mais especificamente, na definição da melhor distribuição de recursos, i.e., estudo do problema de localização, um dos principais enfrentados pelos gestores. Ademais, estudos empíricos defendem que a probabilidade de ocorrência de emergências em regiões urbanas e o nível de risco associado com as mesma está relacionado com a concentração da população. Assim, é importante considerar as diferentes mudanças dinâmicas durante o dia na localização de frota.

A maioria dos estudos realizados previamente fixa o tempo de viagem entre as posições dos veículos na cidade em estudo e, consequentemente, considera as localizações destes fixas. O proposto nesta dissertação é definir localizações flexíveis para veículos de combate de incêndios durante períodos de tempo do dia específicos, tendo em conta as variações no tráfego e a probabilidade esperada de ocorrência de emergências, estimados a partir da migração populacional. Desta forma, foi formulado um modelo de programação linear para posicionar e/ou reposicionar temporariamente veículos de combate de incêndios, maximizando a cobertura de emergências no limite de tempo estabelecido.

Com base em dois problemas diferentes de duas municipalidades da cidade de Santiago do Chile, é demonstrado que esta abordagem melhora o desempenho de cerca de 63% de cobertura para cerca de 97% (chegadas ao local em menos de sete minutos depois do alerta). É também discutida a melhor configuração que garante cobertura total na municipalidade de San Bernardo. Adicionalmente, o modelo é adaptável para a flexibilidade do sistema em estudo em posicionar veículos no exterior das suas companhias, de acordo com especificações legais e operacionais. Apesar de ter sido possível, modelar um problema real, o período de trabalho desta dissertação não permitiu a recolha de informação para calcular o custo operacional associado à aplicação da solução proposta.

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1 Introduction

1.1 Purpose

According to the National Institute of Statistic – INE (June 2015), the Metropolitan Region of Santiago, capital of Chile, houses 7,2 millions inhabitants, representing more than 40% of the total estimated population of Chile. The published predictions point to the maintenance of the fast growth tendency registered in the last 10 years. Thus, the city of Santiago struggles with over congested infrastructures. As emergency response is highly dependent on traffic conditions, the Cuerpo de Bomberos de Santiago (CBS) intents to optimize their network in order to improve the service level and reduce their response time.

This study focus in *Cuerpo de Bomberos de San Bernardo* (CBSB) that is responsible for two municipalities of the capital: *San Bernardo* and *El Bosque*. The province of San Bernardo has a population of around 315.221 over 155 km^2. El Bosque 166.514 in an area of 14,2 km^2, making a total of 481.735 individuals (National Institute of Statistics, in 2012). There is a high probability that there are more people living in this part of the city of Santiago, as the same statistical study , shows an annual increase rate of population in San Bernardo of around 30%, despite the 5% decrease registered in El Bosque.

CBSB relies on eight stations (owned and operated by them) with basic life support,fire fighting and administrative vehicles of different types, in a total of thirty. The usage of vehicles during the years of 2016 and 2017 is represented in the following graphic. Only twenty of this vehicles are operational and have capacity to answer to emergencies, being the other ones used for support services, as for example, transport of medical and paramedical support in case of injured not reported initially or official mobilizations.

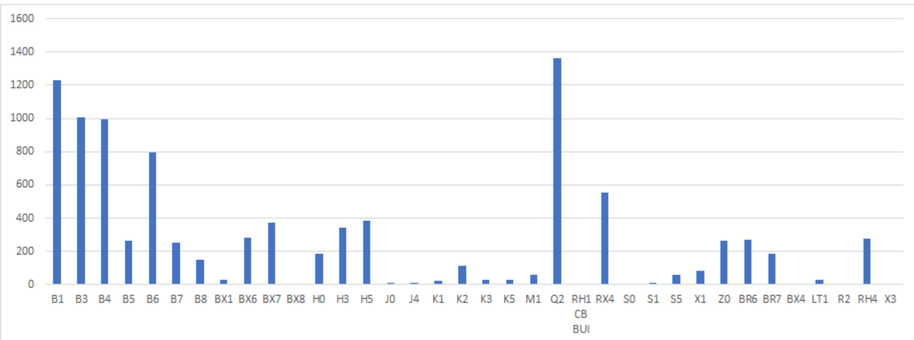

Figure 1: Number of emergencies attended by the registered vehicles for the eight fireman stations during 2016 and 2017, including administrative vehicles

The names of the vehicles contain a code that identifies the type of vehicle and a number that indicates the station to which they are assigned. Vehicles with a zero are temporally assigned to different stations according to maintenance and other operational needs.

According to the type, a vehicle contains equipment to support the population in different kinds of emergencies. In the first appendix, there is a list of the different types of emergencies attended by CBSB with the respective code and

description. Appendix B and C show the types of vehicles and their specifications and the list of CBSB fire stations and their vehicles, respectively.
The location of the stations can be seen in blue icons in the following figure, that provides the satellite view of the municipalities delimited with red lines. The distribution of the fire stations follows historical, political, social and cultural factors and not necessarily efficiency parameters. As can be seen in the satellite view, the residential area is mainly located in the northern part of the municipality what explains the concentration of resources there. Nevertheless, the fire department must operate efficiently in both residential and rural areas.

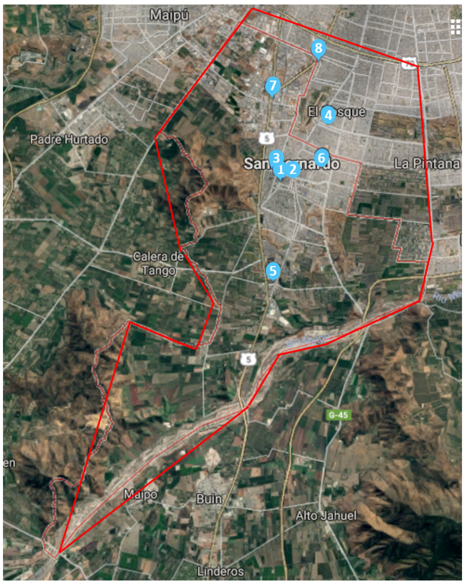

Figure 2: Satellite view of the municipalities of San Bernardo and El Bosque using Google Maps - july 4 2018

Given these resources availability, we aim to determine the optimal temporary location of some of the vehicles in a way that most improves the response time for potential future emergencies in the area of coverage of CBSB.
This dissertation is part of a continuous work with the CBS, the FONDEF IT15I10050- *Respuesta Automatixada para Inteligencia en Despachos Online (RAPIDO).*

1.2 The FONDEF Project

Founded on 1991, the Fondo de Fomento al Desarrollo Científico y Tecnológico (Fondef) supports applied research and technological development of interest for the general public with the purpose of improving Chilean national competition and quality of life. This initiative includes projects with high scientific value that produce favourable social and economic impacts, linking different companies, research institutions and other related entities with a public purpose.
The present research is the second FONDEF involving the *Cuerpo de Bomberos de Santiago* and the University of Chile, under the supervision of Doctor Fernando Ordoñez. Previous work improved the dispatching system by including time dependent travel time estimates, calculated with the speed data provided by the public system of buses, *Transantiago*, and a correcting factor registered by CBS. This

way, the dispatching system uses different speed profiles for different periods of the day, which reflect the traffic changes, so the selected department to answer to a certain emergency might not be the closest one but the one that can reach the local in less time. This system was successfully implemented and has resulted in a reduction of 88.15 seconds in the response time. Consequently, the coverage (response time until 5 minutes) increased from 56,6% to 68,45% using this dispatch rule.

1.3 Problem Definition

In virtue of the exposed previously, the problem that we aim to solve is to identify which are the best locations to position the resources to guarantee the best possible coverage to the two municipalities. This way we have:

1. Demand nodes: where there is a certain probability of occurrence of an emergency.
2. Fixed Locations: locations of fire stations which cannot be moved or closed.
3. Flexible Locations: candidate location where a vehicle can be parked.
4. Vehicle: which can be allocated to either fixed or flexible locations.

Using these elements, we look for a layout of vehicles and fire stations that allows the CBSB to arrive to the highest number of emergencies in less than seven minutes with all the necessary equipment.

1.4 Objectives

The main aim of this research is to determine optional locations of fire vehicles for San Bernardo department. This entails, in practice, the definition of the emergency vehicles locations in different periods of the day according to seasonality of the demand to ensure a target level of coverage for all the territory. The problem is formulated as a general facility location problem, where demand is allocated to each facility taking into account its geographic and time distributions.

The facility location problem is commonly considered a strategic level planning model, frequently related to facilities construction or material acquisition. In this study, we aim to preposition the vehicles as part of the resources allocation activities. This means that the positioning of the resources is time dependent during the day. For example, the same vehicle could be located in a specific corner during a morning and be relocated to another place for the afternoon shift. However fixed for months, i.e., the cycle (of vehicle sites for each period of the day) should be repeated during months until it is noticed a shift in the demand pattern or in the streets' speed profile, as it may happen, for instances, with season changes. This means that the problem must be solved, taking into account a shorter time horizon as well as less capital investment than typical strategic decisions yet not as short as operational as it is not expected to change the vehicles' layout as a daily operation. Hence, the problem will be later treated as a tactical problem.

Overall, the prime aim is to reach the best possible scheduled layout for the emergency response vehicles during daily operations that results in shorter response times using the available equipment and facilities.
At last, an useful tool was to be developed and handed to University of Chile and CBSB that supports the decision making for future strategic and tactical matters.

1.5 Research Strategy

The problem studied in the present thesis is commonly called facility location problem and it is one of the optimization challenges in Operational Research discipline. Thereupon, this study makes use of known techniques to tackle a practical problem.
To start with, a mathematical model was developed to represent the operating modus of CBSB. Additionally, the nature of the challenge required demographic investigation as well as analysis of the previous work done in the FONDEF project that was also conducted at the initial state of this research.
The data provided by the CBSB includes every emergency between July 2015 and March 2018 attended by them, the respective geographical reference (latitude and longitude), date, the exact time of the call reception and the vehicle used.

1.6 Thesis Overview

This dissertation begins with a general contextualization of the facility location problem, focused specially on emergency service systems. The goal of this chapter is to present and point out the characteristics and critique of the main models of this field of investigation. Due to the problem complexity, we discuss the different solution approaches to determine the best known solution for the problem in a subsection.
Next, our theoretical problem formulation is presented in detail.
The forth section presents the two case studies, *SanBernardo* and *Calera de Tango*, which are specific problems solved using adapted variations of our general model. The results of each one are also presented and discussed.
Finally, an overall analysis of the investigation is done with suggestions of work for future research on this type of problems and specific for the practical cases studied.

2 Literature Review

The facility location problem for emergency service applications is a challenge with great significance in operational research. Thus, this problem has been widely studied through different approaches (Caccetta and Dzator, 2013).
To start, emergency refers to an unexpected situation that has to be dealt with as soon as possible or within a defined time limit, and can be provoked by a broad range of causes, as for example, natural disasters, accidents or malfunctioning equipment. The present investigation focus on everyday fire emergencies, in which the mobile equipment, such as fire fighting vehicles, is deployed to the local, where the danger to people or property is, the soonest possible. Here we do not take into account large-scale emergencies, such as hurricanes, earthquakes or explosions, nor the fixed service centres, such as hospitals or trauma centres, that imply the dislocation of the injured individuals (Marianov, 2017). In an everyday setting, medical, fire, crime or equipment failure emergencies happen mostly isolated or a several at a time yet independently and are characterized to be hard to predict(Marianov, 2017). This way, the positioning of servers is crucial to arrive the soonest possible to all the emergencies sites.

2.1 Solution Approach

The location problem can be solved using two kinds of methods:

i. Descriptive methods .
Iterative methods start by guessing a set of locations, then evaluate the performance of the system, followed by the change of locations in a smart way and new evaluation of the performance. This steps are repeated until a stopping criteria is met. The performance of the system can be estimated either with simulation or using a descriptive model. The trouble is that these procedures require a long interval of time and large amounts of data to find a solution (Marianov, 2017). Often, simulation is used to assess the system performance. Although descriptive models are able to more clearly represent how the real system works.

ii. Prescriptive methods .
This approach defines a set of locations that optimize a establish criteria, using either mathematical programming or heuristic algorithms. This models lack of the realism of the descriptive models, however, they are able to find solutions in a fraction of the time required by the previous (Marianov, 2017).There exist several heuristics and metaheuristics used to solve the problem using either mathematical programming or heuristic methods. The main ones used for location problems are reviewed in the next paragraphs.

Genetic algorithm .
GA is an optimization approach inspired in nature and has been vastly applied in operational research problem optimisation. The main characteristic is the improvement of generations through reproduction mechanism, which include crossover and mutation. Thus, each iteration involves selection of a set of parents, based on their fitness value and application of reproduction schemes to

generate a set of offspring. Each generation consists of a set of individual solutions and a selection strategy is used to update the population. This process is repeated until the termination criterion is met (Zarandi et al, 2011).

Tabu search heuristic .
Tabu search is a local search method also based on a natural process. The main characteristic is that a list of solutions, which are declared as tabu (forbidden) points for a number of consecutive iterations, is created and functions as memory. This mechanism avoids being trapped in local optima and consequently the algorithm moves at each iteration from a solution to one of its neighbours even if it causes a deterioration of the value of the objective function (Michalewicz and Fogel, 1998).

Locate-allocate .
Locate-allocate heuristics are design specifically for location problems and solve not only the location problem but also allocate the demand to servers. This method divides the main problem into two simpler ones, taking advantage of the fact that the separate phases, i.e. locating and allocating, are each easier to solve than the combine problem. Thus, this approach is widely used to solve this type of problems as locate-allocate heuristics usually provide a good solution within a relatively short computational time (Jia et al, 2007).
A limitation of this methods is that normally each demand point is serviced by only a single facility, which is not always true when it comes to emergency assistance. However, changes can be made to enable the heuristic to deal with the location problems with multiple facility quantity-of-coverage and quality-of-coverage requirements, as done by, for example, Jia et al (2007).
Generally, the quality of the final solution achieved and the algorithm convergence are highly dependent on the initial solution given when using a prescriptive method. Thus, this heuristics are often used, combined with good quality constructive heuristics.
As emergency service systems face stochastic demand with variable distribution in time, the problem can be non-linear and time variant (Repede and Bernardo, 1994). Henceforward it is required the use of heuristics as a solution method to achieve a solution within a reasonable period of time.
According to Zarandi et al (2011), genetic algorithms (GA) has been proven to be one of the best methods to solve facility location models with better performance when compared with local search procedures. Also, Gendreau et al (2000), concludes that the use of the tabu search heuristic for real-time ambulance relocation is only serviceable when associated with parallel computing, taking advantage of the vacant time between consecutive calls and predicting future scenarios. Moreover, the developed tabu search algorithm is said to be adequate for the static location problem.
In this research, we solved the mathematical programming formulation using exact optimization models, i.e., a mixed integer optimization solves. Consequently, in the next section we discuss the different mathematical programming formulations for the typical emergency services location problems.

2.2 Mathematical Programming Formulations

The most common objective is the coverage of the demand areas, as the safety and well-being of the community are highly dependent on the response time of fireman and other emergency entities (Jia et al., 2007; Gendreau et al, 2000). More specifically in the fire fighting context, Challands (2010) concludes that the amount of structural damage increases with the response time of fire services. Thus, ensuring sufficient coverage is a vital restriction of the challenge of the present investigation.
However, according to Zaffar et al. (2016), survivability objectives tend to produce better coverage results than the maximum coverage objective. Their results indicate that emergency service systems (ESS) can benefit from the use of survivability models in planning decisions such as determining the staging locations of their fleet. Nonetheless, survivability statistics are difficult to collect as estimating patient survival is problematic since it can be assessed at the hospital and patients can be discharged several days after the emergency. Also, the failure can be responsibility of this post delivery care. Second, patient survival information is not readily available due to medical privacy regulations. Third, the patient data and the formal processes that the use of such kind of information make the use of the survival rates as an estimative of the performance of the emergency systems impractical.
Hence, for this study, we considered the coverage as the main efficiency measured, taking into account the required maximum response time indicated by the professionals of CBSB of seven minutes.
There are three main types of mathematical models reported in the literature specifically for this type of problem. They will be exposed in the next divisions of this section.
Yet one must define the main indicators of the service level for emergency service systems that are important to the comprehension of the models specificities. Those indicators are as follows:
(a) Dispatch delay time: time interval between the receipt of the call and the assignment of an vehicle as defined by (Repede and Bernardo, 1994). Nevertheless, this is mainly a technical and operational factor that will not be analysed for optimization purposes in this study.
(b) Response time: amount of time between the reception of a call and the arrival of the emergency vehicle. This is the main indicator for most of the optimization models and likewise in the conducted study.
(c) Total service time: period of time spent to provide the entire service, including victims' transportation to the hospital, if needed, and vehicle available, even if still in route to the respective base.
(d) Loss rate: fraction of calls that were not answered within the response time predefined standard (often referred as coverage standard or time standard).
(e) Success rate: on the contrary, this indicator is the fraction of calls that are reached within the standard time.
Ultimately, the most relevant mathematical models to the proposed dilemma assist managers to utilize the accessible emergency response vehicles to provide the best possible response time to emergencies over all the population (Jagtenberg et al, 2015). That said, the existing models are the following:

i. P-centre models(minimax) .
In brief, p-centre models locate a defined number of facilities by minimizing the maximum distance between these facilities and the demand nodes assigned to them (Garfinkel et al,1977). Also commonly called minimax, a p-centre model was used by Talwar (2002) to solve a problem concerning helicopters location to answer emergencies in a touristic area of the Italian Alps, where the slowest time to arrive to the emergency local was minimized. At last,p-centre models optimize the network by improving the weakest link.
One of the troubles with this type of models is that fixing a pre-defined number of servers may result that the amount of time needed to arrive to the most remote locations can still be unacceptable after the optimization (Marianov, 2017).

ii. P-median models(minisum or minimum average) .
Another way to estimate the efficiency of an emergency system network is based on the average distance between demand node and the facilities locations, as its decrease represents a decrease in the average response time. The p-median models determine the location of p facilities so that the average distance between demand nodes and their closest facility is minimized (Caccetta and Dzator, 2013). One must highlight that this model, antagonistically to the exposed previous, improves the global performance of the network by minimizing the average. This fact is one of the weak characteristics of this model because the difference between the shortest and the longest emergency-server distance can be very significant and consequently, it is possible that, in some cases, the amount of time needed to arrive to the most remote locations is also unacceptable, as happens with minimax models (Marianov, 2017).

iii. Covering Models .
This type of models address coverage, which means that the facilities are placed so that any demand node will be attended in a period of time smaller than the defined standard time or place closer than a pre-determined distance, say 10 minutes or 2 kilometres. This way, the demand is treated equally and is only considered covered if a vehicle can attend to the local within the standard time, or is located in a node closer than the distance limit (Marianov,2017). Treating the demand equally means that there's no importance given to the emergencies' spatial concentration as it is equally important to cover a node with very low demand as it is to cover a node with high demand.
Covering models have similarities with the p-median centre as the main objective is to improve the overall systems performance (Boonmee et al., 2017), and not specific linkages of the system as p-center models. One must also highlight that, commonly, the coverage criterion is complemented with other criteria, as for example, cost, balanced work of all employees of the service and others (Marianov, 2017). The investigation within this approach led to a divergent evolution of models usually categorized in two groups: localization set covering models (LSCM), introduced by Toregas et al. (1971) and maximal covering location problems (MCLP), described by Larson (1973).
The main difference between the two categories is that MCLP maximizes the portion of demand covered assuming there are N servers while the LSCM aims to minimize the number of facilities needed to ensure coverage for every demand

node.
A limitation of this type of models is that, being the location variable binary, only one ambulance is placed at each site and so coverage requires only one ambulance, excluding double, triple or multiple coverage (Marianov, 2017).

a. Maximal Covering Location Problems .
Several variations and extensions to the maximal covering location problem (MCLP) have been conducted since the very first presentation of this approach by progressively adding complexity to represent the diverse factors that occur in reality. One example is Repede and Bernardo (1994) with the TIMEXCLP model that was developed by incorporating temporal variation in the daily demand process in addition to spatial variation and multiple states of vehicle availability. The goal was to maximize the expected covered demand more accurately than in previous researches by avoiding the assumption that all vehicles are available at alltimes, common in previous models. Ball and Lin (1993) proposed a reliability model that refines the previous formulations by taking into account a failure rate related to vehicles availability and guarantees that each demand point receives at least the required service level. The main aim is to assign the owned vehicles to the predefined candidate location for fire stations by minimizing the fixed cost incurred to build new facilities or renovating an existing one and the variable portion of housing a certain number of vehicle at the different stations.

b. Localization Set Covering Models .
Regarding the localization set covering model's line of investigation, the idle localizations are selected by minimizing the total number of facilities while covering all the demand. The rationale behind the minimization of the number of facilities relies on the assumption that the costs are identical for any candidate localization and thus, the number of facilities can be used as a proxy of costs (Toregas et al., 1971). This is not necessarily so as Marianov (2017) points out, but, in the particular problem in study of parking some specific vehicles outside the fire stations for determined periods of time, the differences of costs between the candidate locations are insignificant. Rajagopalan et al (2008) formulated the dynamic available coverage location model (DACL) which determines, with a predefined reliability, the minimum number of ambulances and the respective locations for each time cluster, within which the demand pattern takes significant changes, while respecting the coverage requirements. This models rely on the hypothesis of an unlimited budget that is not the case, in most real problems. Also, the formulation does not distinguish the different types of vehicles and the corresponding ability to answer to specific emergency types (Marianov, 2017). Fact that can be of big importance in some cases.

Contribution .
In order to overcome the previously discussed drawbacks, we propose a model similar to a MCLP with limited resources and aiming to cover the maximum demand possible but, at the same time, integrates the variation of the emergency demand and travel time, distinction of the different types of emergencies that require different combat equipment and both flexible and fixed vehicle locations. Through the simultaneous consideration of different dynamic aspects and

operational specifications, the resources can be temporally differentiated leading to an efficient use of resources. Thus, the proposed model supports small municipalities for locating emergency support vehicle on a tactical decision level.

3 Problem Definition and Model Formulation

3.1 Definition of the demand nodes inside the municipalities

Although, in exceptional conditions, it is possible that the CBSB is called to answer emergencies situations outside their coverage area, the layout of the resources should not take into account demand nodes outside the municipalities. This way, it was necessary to define the nodes inside the coverage area.
The challenge at this stage is the low information about the jurisdiction limits of the unit, and so, the limits were defined using the information registered in google maps, the results obtained in the previous research and the data relative to the historical calls attended by the CBSB. The unit used was the geographical system with six significant decimal digits which permits a accuracy of eleven centimetres. There was obtained a list of nodes of the type: (node id, position x, position y, emergency probability).

3.2 Location of the fireman stations

The correct location of each one of the eight fire stations of CBSB is vital to represent more accurately the actual distribution of resources and better estimate the expected coverage of the two municipalities. In this problem, we used geo-referencing of all the fire stations. Additionally, to solve the problem it was necessary to define the resources which are actually assigned to each fireman station. This information was obtain by contacting CBSB and visiting their official websites. There was obtained a list of fixed facilities of the type: (fireman station id, position x, position y, number of vehicles assigned).

3.3 Location of flexible sites

The definition of the flexible location was done differently for each specific case study, according to the information available. There were used three distinct alternatives:

- Candidate sites indicated by the CBSC.
- Candidate locations determined according to the coverage level of all the nodes that represent the municipality.
- Considering all the demand nodes as possible flexible locations.

For each case study, there was selected the most appropriate alternative.

3.4 Definition of the travel time table

Instead of using a single travel time table which would contain the expected travel time between each fire station and every demand node, we decided to have distinct tables during the day that better represent the traffic. In the tool used to dispatch the vehicles for each emergency, there were determined the speed profiles for every half hour of the day and so the day was divided in 48 time periods. Analysing this data, we concluded that the times are often repeated existing only around 5 different speed profile. For simplicity, the time periods are an equidistant segmentation of a twenty four hours day into four different periods, which represent the changes in the studied speed profiles. Each time period t begins at point t in time, lasts until $t + 1$ and has the duration of Δt.

Table 1: Segmentation of the day used

Period Index t	Hour Interval
1	0 - 6
2	7 - 12
3	13 - 18
4	19 - 24

The final value of each period is the maximum time of all periods of thirty minutes contained in each interval for each facility and each candidate location to each demand node.

3.5 Definition of the types of vehicles

To solve the problem, we will only take into considerations two types of emergencies that require firemen action in the first seven minutes: type 1 that includes codes 10-1 to 10-16 and type 2 that is a direct correspondent of code 10-0 structural call.

3.6 Definition of the types of vehicles

It is important to understand the available equipment and the respective characteristics. To solve the problem, we focused on the two main types of vehicles required for rescue activities and they are the following:

- type B or BX: vehicles equipped with a motor to bomb water and that can transport water to the emergency - type A in our formulation;
- type Q: vehicles equipped with extendible stairs for rescuing - type B in our formulation;

Each fire station has its own vehicles. This resource allocation is detailed in the appendices.

3.7 Model

The problem is formulated, following the logic of the time-dependent vehicle allocation model proposed by Degel et al (2013), but considering the locations of two different types of vehicles and two distinct types of emergencies.
Nevertheless, there are assumptions needed to make the problem more tractable. Let J be the set of all potential vehicle locations, that can be fixed existing facilities or specific street location pointed out by the preliminary analysis. This set is defined by the following subsets: W is the set of existing fire stations and F are the street location candidates (outside the fire stations).

The set of demand nodes is denoted by D. Each demand node $i \in D$ is associated with the corresponding demand value for each type of emergency, d^1_{it} and d^2_{it}, which contain the expected demand in node i at time t for the type 1 and 2, respectively.
The travel time matrix is also an exogenous parameter stored at the matrix tt_{jit} that contains the expected travel time between the vehicle location j and the demand node i during the period of the day t. The total number of emergency vehicles of the type A and B is denoted by M_A and M_B,respectively, while $\mathrm{N} \in \mathbb{N}$ is the number of demand nodes.
The capacity of an emergency vehicle of type A is c^A, which is calculated by dividing the average time that is needed to serve one emergency by the length of the periods and so, is given by $\mathrm{c} = \bar{t}/\ \Delta\mathrm{t}$. The same applies for vehicles B.
As mentioned earlier, for a demand node to be considered covered there has to be an available vehicle in a radius of seven minutes of travelling time to arrive in case of an emergency. The binary decision variable y^1_{jt} indicates whether the emergencies of type in site $i \in D$ are covered in time period t, e.g. y^1_{jt}=1 and y^2_{jt}=1, for type 1 and 2. When it comes to the minimum coverage requirements, these are distinct for each type of demand and are described further with the respective constraint.
The second and third decision variables $x^A_{it} \in \mathbb{N}_0$, $x^B_{it} \in \mathbb{N}_0$ indicate the number of vehicles of the type A and B, respectively, which are located at node $i \in J$ during period t.
As we are assigning vehicles to locations for periods t of a day, locations are repeated daily, being the four periods a cycle, which means that at the end of the period 4, a day is over, and the vehicle must be moved to the location where it is needed at the next period 1 (of the next day). This way, it is important to determine this movements of vehicles to better estimate the cost of the solution.
The number of vehicles which are relocated from a node $j \in J$ to a location $k \in J$ between period t-1 and t, for $t \in T\backslash\{1\}$, is denoted by $\mathrm{u}_{jkt} \in \mathbb{N}_0$. For instance, the variable u_{jk1} indicates the number of vehicles that are relocated after period NT and before period 1 of the next day.
The primary goal of this optimization is to maximize demand coverage while taking into account the degree flexibility provided by the organization on the use of flexible locations and relocations. Hence, there is a trade-off between these goals regarding flexibility (use of relocations and flexible locations) and practicability (operational constraints). For example, there are costs associated with positioning vehicles outside the fire departments, as it involves material transportation, staff mobilization and time. Also, the work conditions and comfort of the personal is compromised if any site of the municipality can be considered as candidate location for the vehicle as there is no support facility. Hence, for organizational/operational reasons, relocations and use of flexible locations are to be used carefully and avoided if do not incur in significant improvement of the final coverage. The consideration in the objective function allows the decision maker to control this trade-off. To do so, the most appropriate multi-criteria approach is the weighted sum, the vector λ is a vector of coefficients $(\lambda_1, ..., \lambda_L)$ greater than 0 for all $l \in L$. A list of all the variables and parameters can be consulted in appendix D.
The three objectives are the following: (2) covered demand, (3) and (4) the number of relocations of emergency vehicles for the kinds of vehicle A and B,

(5) and (6) the number of vehicles outside the official facilities of each type. The cost of relocation and the difficulties associated to position a vehicle of the two types of vehicles are not the same.

$$\max \sum_{l=1}^{5} \lambda_l \cdot z_l \tag{1}$$

$$z_1 = \sum_{t=1}^{NT} \sum_{i=1}^{N} d_{it}^1 \cdot y_{it}^1 + d_{it}^2 \cdot y_{it}^2 \tag{2}$$

$$z_2 = -\sum_{t=1}^{NT} \sum_{j \in J} \sum_{k=1 \in J} u_{jkt}^A \tag{3}$$

$$z_3 = -\sum_{t=1}^{NT} \sum_{j \in J} \sum_{k \in J} u_{jkt}^B \tag{4}$$

$$z_4 = -\sum_{t=1}^{NT} \sum_{j \in F} x_{jt}^A \tag{5}$$

$$z_5 = -\sum_{t=1}^{NT} \sum_{j=1 \in F} x_{jt}^B \tag{6}$$

$$\sum_{j \in J, tt_{jit} \leq 7} x_{jt}^A + \sum_{j \in J, tt_{jit} \leq 7} x_{jt}^B \geq y_{it}^N, \forall i \in D, \forall t \in T \tag{7}$$

$$\sum_{j \in J, tt_{jit} \leq 7} x_{jt}^A \geq y_{it}^2, \forall i \in D, \forall t \in T \tag{8}$$

$$\sum_{j \in J, tt_{jit} \leq 7} x_{jt}^B \geq y_{it}^2, \forall i \in D, \forall t \in T \tag{9}$$

$$\sum_{i=1}^{N} d_{it}^1 \cdot y_{it}^1 \leq \sum_{j \in J, tt_{jit} \leq 7} \sum_{i=1}^{N} x_{jt}^A \cdot c^A + \sum_{j \in J, tt_{jit} \leq 7} \sum_{i=1}^{N} x_{jt}^B \cdot c^B, \forall t \in T \tag{10}$$

$$\sum_{i=1}^{N} d_{it}^2 \cdot y_{it}^2 \leq \sum_{j \in J, tt_{jit} \leq 7} \sum_{i=1}^{N} x_{jt}^A \cdot c^A, \forall t \in T \tag{11}$$

$$\sum_{i=1}^{N} d_{it}^2 \cdot y_{it}^2 \leq \sum_{j \in J, tt_{jit} \leq 7} \sum_{i=1}^{N} x_{jt}^B \cdot c^B, \forall t \in T \tag{12}$$

$$\sum_{j=1}^{N} x_{jt}^A \leq M^A, \forall t \in T \tag{13}$$

$$\sum_{j=1}^{N} x_{jt}^B \leq M^B, \forall t \in T \tag{14}$$

$$x_{jt}^{A}+\sum_{k\in J}u_{kj(t+1)}^{A}-\sum_{k\in J}u_{jk(t+1)}^{A}=x_{j(t+1)}^{A},\forall t\in T\setminus\{1\},\forall j\in J \tag{15}$$

$$x_{jNT}^{A}+\sum_{k\in J}u_{kj1}^{A}-\sum_{l\in J}u_{jk1}^{A}=x_{j1}^{A},\forall j\in J \tag{16}$$

$$x_{jt}^{B}+\sum_{k\in J}u_{kj(t+1)}^{B}-\sum_{k\in J}u_{jk(t+1)}^{B}=x_{j(t+1)}^{B},\forall t\in T\setminus\{N\},\forall j\in J \tag{17}$$

$$x_{jT}^{B}+\sum_{k\in J\cup A}u_{kj1}^{B}-\sum_{k\in J\cup A}u_{jk1}^{B}=x_{j1}^{B},\forall j\in J\cup A \tag{18}$$

$$x_{jt}^{A},x_{jt}^{B}\in\{0,1,...,M\}\subset\mathbb{N}_0\forall j\in J,\forall t\in T \tag{19}$$

$$u_{kjt}^{A},u_{kjt}^{B},u_{jkt}^{A},u_{jkt}^{B}\in\{0,1,...,M\}\subset\mathbb{N}_0\forall j\in J,\forall t\in T\forall j\in J \tag{20}$$

$$y_{it}^{1},y_{it}^{2}\in\{0,1\}\forall i\in D\forall t\in T \tag{21}$$

$$z_l\in\mathbb{R} \tag{22}$$

Constraints (7) ensures that the expected demand of type 1 in the node j is only considered covered if there is at least one vehicle, of any type, is located within a 7 minutes radius of the site. While constraint (8) and (9) force the coverage of the demand of type 2 to require, at least, one vehicle of each types, A and B, 7 minutes away from the emergency site.
Constraints (10) to (12) are capacity constraints, i.e., guarantee that for each period of time t, the overall capacity of vehicles is sufficient to cover the total quantity of emergencies to attend. Constraints (13) and (14) limit the total number of vehicles in each period t.
When it comes to the reallocation constraints,the equations (15) and (17) are flow constraints that count all the reallocations of vehicles, for type A and B, respectively. (16) and (18) make the connection between the last shift of one day and the previous of the next as the service is provided continually, i.e., 24 hours per day and, as exposed previously, the location are defined as a daily cycle.
Equations (19) to (22) define the variable ranges.
For the restriction (7), in case the software used does not allow to imply conditions, can be done using subsets of J.
As the nature of each objective is significantly different the coefficients lambda were defined as an economic value: for (2) expected income resultant of the coverage of an emergency and for equations (3) to (6) estimated cost associated.

4 Application and Case Studies

4.1 San Bernardo and El Bosque

To properly identify the problem and the service provided at the moment by CBSB, we had access to historical data from July 2015 to March 2018: a total of 9653 emergencies. In figure 3, there is an image of how the data was provided.

	A	B	C	D	E	F	G	H	I	J	K
1	correlativo_uni	Clave	carro	Calle	Comuna	Esquina	Fecha_Despacl	Fecha_Alarma	latitude	longitude	Hora_Alarma
2	2018-20214389	10-9-6	H0	AVDA PORTALES	SAN BERNARDO	AVDA PORTALES ORI	22-03-2018	22-03-2018	-33,6310011828260	-70,7038605597240	9:40:27
3	2018-20214387	10-2-1	H5	CAMINO EL BARRAN(	SAN BERNARDO	AVDA ELIODORO YAI	22-03-2018	22-03-2018	-33,6691955587020	-70,7436771154149	8:04:10
4	2018-20214386	10-2-1	B3	CAMINO EL BARRAN(	SAN BERNARDO	CALLE CAMINO ROMI	22-03-2018	22-03-2018	-33,6936616103145	-70,7448940576358	3:51:57
5	2018-20214385	10-7-1	B4	CALLE LOS CIPRESE!	EL BOSQUE	AVDA LO MARTINEZ	21-03-2018	21-03-2018	-33,5691818033147	-70,6802161362930	19:46:48
6	2018-20214384	10-2-1	B3	CALLE SAN JOSE	SAN BERNARDO	CALLE DUCAUD	21-03-2018	21-03-2018	-33,6031195602654	-70,6859609799584	16:32:21
7	2018-20214383	10-2-1	B6	CALLE SAN MARTIN	SAN BERNARDO	CALLE BALMACEDA	21-03-2018	21-03-2018	-33,5866587931825	-70,6947562076774	8:42:40
8	2018-20214382	10-3-14	Q2	CALLE CAMINO DE N	SAN BERNARDO	CALLE LOS SUSPIRO	20-03-2018	20-03-2018	-33,6403800859667	-70,6966276151277	22:49:27
9	2018-20214381	10-4-1	B1	CALLE COVADONGA	SAN BERNARDO	CALLE URMENETA	20-03-2018	20-03-2018	-33,5941052351584	-70,7073909884140	21:52:08
10	2018-20214381	10-4-1	Q2	CALLE COVADONGA	SAN BERNARDO	CALLE URMENETA	20-03-2018	20-03-2018	-33,5941052351584	-70,7073909884140	21:52:08

Figure 3: Data base with the historical emergencies of CBSB.

From each registry we extracted the relevant data for the study: geographical reference, latitude and longitude of the emergency, time of occurrence, type of emergency and vehicle dispatched. Time data had precision to the nearest second. With this, a simpler and more tractable data base was constructed which permit the analysis presented in the next section.

4.1.1 Preliminary Data Analysis

Through the following analysis, we work with the number of calls in each hour instead of their times of occurrence, to facilitate the application of time series models. In average, there are 10 daily emergency calls to be answered that require the presence of at least one vehicle. The next figure provides an overview of the data: it shows the daily volume of 2016. The figure suggests larger volume in November, December and January, which correspond to summer, a season that holds a higher risk of fires, and shows some unusually large values, e.g., on January 1 and on April 17. On $17^{th}$of April, a unexpected event of intense rain provoked the burst of the river *Mapocho* that crosses the city of Santiago, so it was considered an outlier. When it comes to special days, as for example commemorative dates, the data does not suggest any effect or increase in such events.

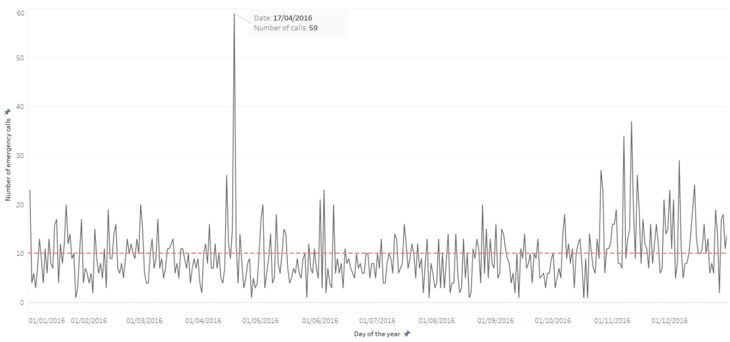

Figure 4: The daily call volume for the year 2016.

Figure 5: The monthly call volume from January 2016 to December 2017

Analysing the second plot, there is a remarkable increase of emergencies during November, December and January and suggests a light positive trend; the likely explanation is the population growth. In February 2017, CBSB has an exceptional low number of emergencies calls explained by state of national emergency, *Alerta Roja*, due to the occurrence of a catastrophe, in the neighbour region of *Valparaíso* that started in the 2^{nd} for February and was only cancelled in March, being the fire extinct at the end of February (ONEMI, 2017). This event caused a mobilization of the company to this region to support the combat activities. In the occurrence of a large scale emergencies, the several fire units in the city or even of the entire country cooperate with one another, and so, there won't be simulated large scale emergencies. Moreover to evaluate the performance and execute the optimization of the resources utilization, the data relative to unusual events will be disregarded.

Figure 6 shows average volume by hour over the week cycle. The plot reveals a clear hour-of-day cycle: over a 24-h cycle, higher call volumes are usually observed between 12 a.m. and 10 p.m.; substantially lower volumes are seen overnight. Also, there is no significant effect of seasons on the daily cycle, which means that the distribution of occurrences during the day is constant throughout the year.

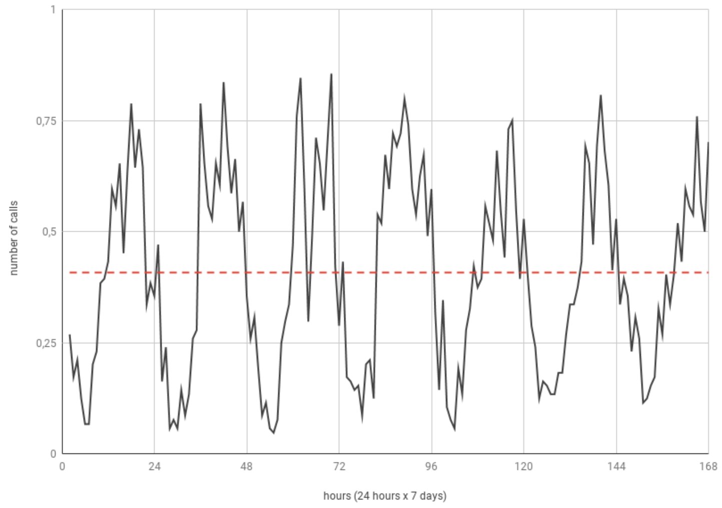

Figure 6: The average hourly call volume over the weekly cycle

Figures 7 and 8 contain box-plots of the daily volume for each day of the week and monthly volume for each month of the year, respectively. Each box plot has two bars located at a distance of 1.5 times the interquartile range(the heigh of the box) below the first quartile and above the fourth quartile. The observations that fall above or below of this bars are marked with small circles. There is no significant difference between the different days of the week, having Thursday only a slight higher volume than the rest of the days. November, December and January are the month with higher number of emergency calls while February and March seem to have the lowest numbers of observations. February is the summer vacation of the majority of the educational centres which leads to a significant emigration of population leaving the city.

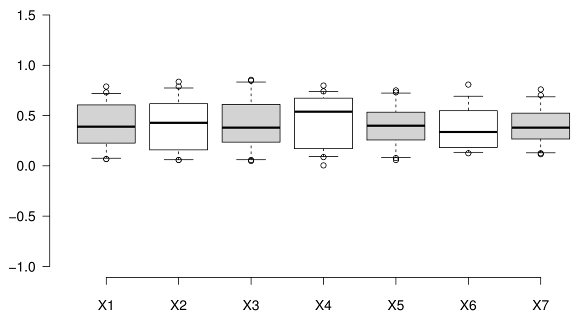

Figure 7: Box plots of arrival volumes per day for each day of the week

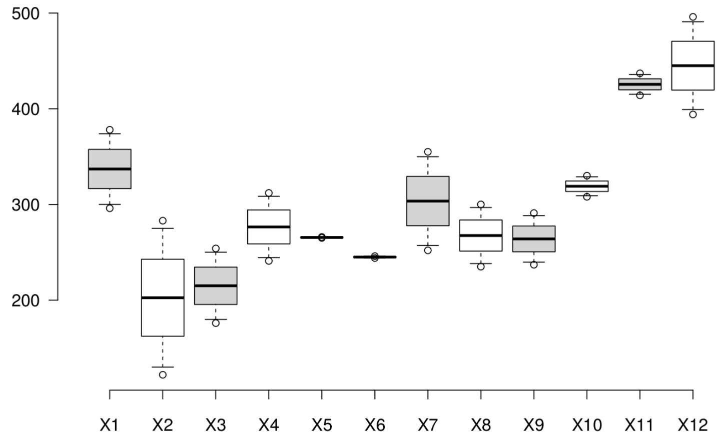

Figure 8: Box plots of arrival volumes per month for each month of the year

In the following figure, one can see the distribution of calls per type of emergency. It is observable that there are mainly registered calls correspond to structure, vehicle, trash and emergency rescue. Only the code 10-0 (structure calls, i.e. fire damaging structural elements of a building) requests the presence of two vehicles to provide support, i.e., this would be the proportion of demand emergencies of type 2 in the model formulation studied previously.

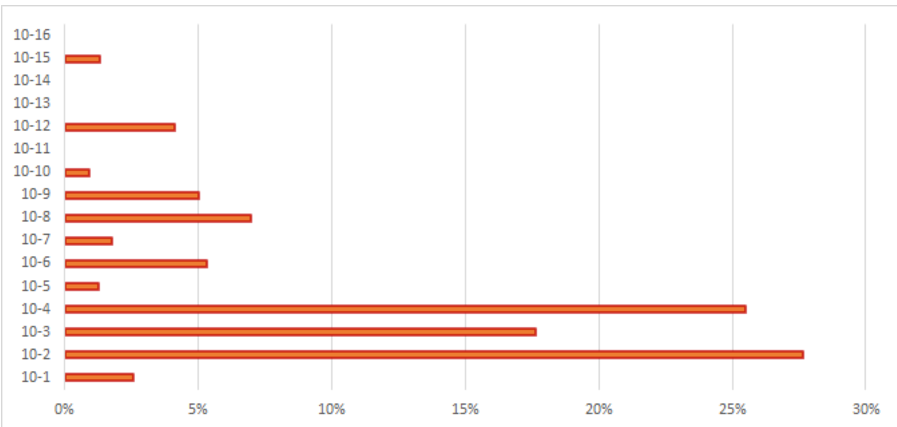

Figure 9: Emergency historical calls distribution by type of call

When it comes to the geographical distribution, the data provided does not support the existence of variation during the different trimesters. In the following map, the emergency sites of all the historical demands in our database are shown.

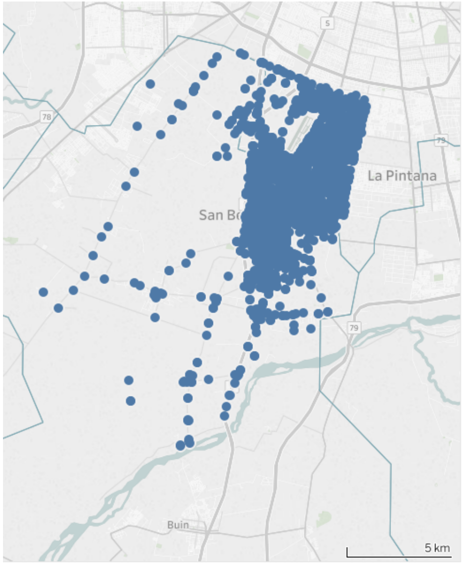

Figure 10: Sites of historical emergencies between July 2015 to March 2018

We can see that as expected the emergencies are concentrated in the residential area where previously, we saw that there exist a higher number of fire stations. To evaluate the coverage capability of the current locations (considering infinite capacity and availability at all times), we created a map with the different covered levels, i.e. the number of deployment stations located within a distance that can be travelled in seven minutes or less, for each and all the nodes used to represent the region.

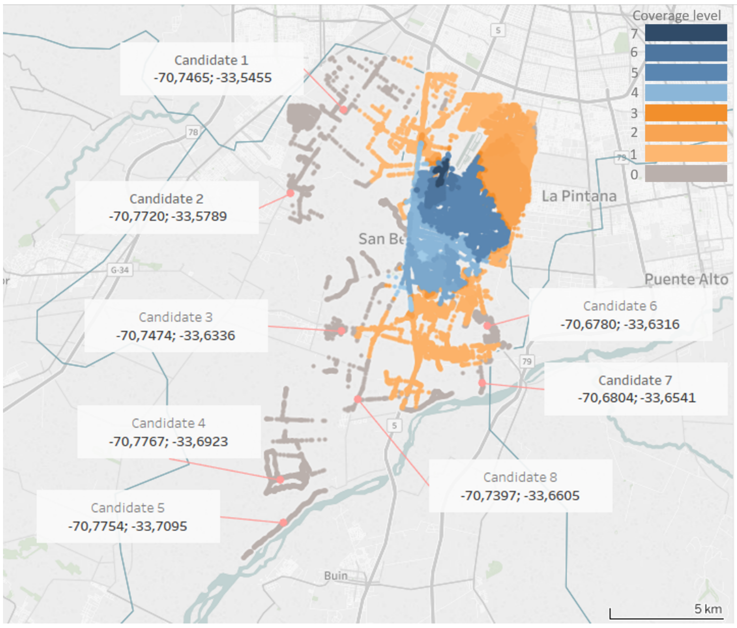

Figure 11: Coverage level of all nodes considered to approximate the communes of San Bernardo and El Bosque

We determined the candidates locations taking into consideration the zones with a coverage level equal to zero (grey points in the map). To better optimize the model it is of crucial importance to take into account capacity restriction as the resources are limited. This way, we considered that it would be pertinent to develop a model to predict the demand level for further applying capacity constraints.

4.1.2 Forecasting Model

Taking into account the low quantity of information available (two complete years) and the characteristics of the arrival process (from many small and independent sources), we considered to perform the forecast at a high aggregation level, i.e., to predict the number of emergency calls per day of all types and from all the nodes of the municipality.
Regarding the outliers identified previously, adjustments were made to February 2017 using a naive ex post forecast, considering the two closest homologous periods. Given the preliminary analysis, we used the mixed model of the Holt Winters method which considers a positive trend and the yearly seasonal cycles with effect of the month of the year, with period s=12, one step ahead. This way, it can be said that:

$$Z_t = \vartheta_t \varphi_t + \varepsilon_t \tag{23}$$

$$\vartheta_t = \vartheta_{t-1} + \tau_{t-1} \tag{24}$$

Where Z_t is the number of arrivals on day t and the parameters $\vartheta_t, \varphi_t, \varepsilon_t, \tau_t$ are real-valued constants. We assume that the residual E_t are independent and identically distributed normal random variables with mean 0 and $\hat{\sigma}_{Et}$. The forecasting model is as follows:

$$\hat{Z}_t(1) = (n_t + b_t) \times f_{t+1-s} \tag{25}$$

Being $n_t = \hat{\vartheta}_t$ an estimate of the level of the series in instant t, $b_t = \hat{\tau}_t$ an estimate of the trend of the series and $f_{t+1-s} = \hat{\varphi}_{t+k}$ an estimate of the seasonal component for instant $t + 1$ (one step ahead). To determine this three basic components, the Holt Winters model utilizes simple exponential smoothing (Kotsialos, 2005). The equations that are applied to each t are:

$$n_t = \alpha \frac{Z_s}{f_{t-s}} + (1 - \alpha)(n_{t-1} + b_{t-1}) \tag{26}$$

$$b_t = \beta(n_t - n_{t-1}) + (1 - \beta) \times b_{t-1} \tag{27}$$

$$f_t = \gamma \frac{Z_t}{n_t} + (1 - \gamma) f_{t-s} \tag{28}$$

To avoid redundant parameters, we impose the following constraints:

$$\alpha, \beta, \gamma_k \epsilon \, [0, 1] \tag{29}$$

The parameters for the regression model (26)-(28) were estimated using least squares technique (see following table):

Table 2: Parameter estimates for the forecasting model

α	β	γ
0,5358853	0	0,9460553

Table 3: Measure Accuracy

(a) Standard Statistical Measures

ME	MAE	MSE
2,766368	49,730650	4803,218075

(b) Relative Measures

MPE	MAPE
-39,64%	16,84%

After, we performed a test to the expected value of the errors. As the sample is small, it was used the distribution t-student to determine the proof value.

$$H_0 : \mu_E = 0 \tag{30}$$

$$H_0 : \mu_E \neq 0 \tag{31}$$

The results obtained (the test statistics is smaller than the proof value) lead us to not reject the null hypothesis and so, the expected value of the errors can be approximate by zero. The test to the decomposition there is not significant correlation between consecutive errors, as the error autocorrelation coefficient is inside the interval given by the critical value, as shown in table 5. Finally, the distribution of calls thorough the day is obtained by diving the total by the number of days of each month, obtaining this way the daily average number of calls and the residuals obtained are displayed in the next figure.

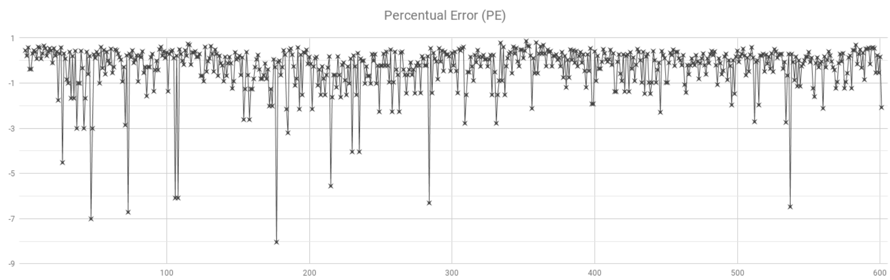

Figure 12: Percentage residuals for the daily forecast

The variability of the errors represented in the previous figure is significantly high, but it can be explained by the fact that emergencies occurrence depends in several demographic characteristics of the population and has an important component of individual behaviour that is not simple to model (and it is not the focus of our investigation). Further, we apply the hourly distribution of the calls, but as the observations per hour can be really low, there won't be done any error analysis.

Table 4: Decomposition Validity

(a) Error Autocorrelation Coefficient

$Critical Value$	r_1
$\pm$ 0,42770706	0,15652080

(b) Comparison with Naive

U-theil
0,79194763

(c) Test to the Expected Value of the Errors

$t_{N-1}(95\%)$	TS
1,72471824	0,17865085

4.1.3 Model Application

There are eight candidate locations which are represented in the following figure. There are also represented all the nodes which were taken into account in the optimization model and the respective level of coverage, i.e., the number of fire station located seven minutes away from the site. At maximum a node is covered by seven of the eight deployment facilities, represented in dark blue. The candidates are positioned close to the regions where there is a concentration of grey points, i.e., nodes that are not covered by any of the existing facilities within the seven minutes requirement.

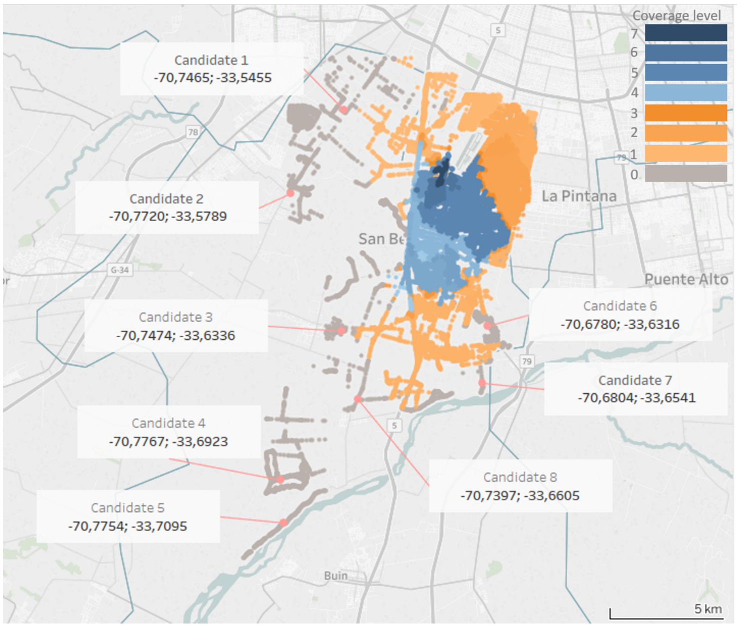

Figure 13: Coverage provided at the moment of the beginning of the present study

At the first stage of this investigation, we were told that all the existing firemen stations should be considered fixed and operative and only one extra locations should be determined. This way, we made several changes to the model presented previously to adapt to this specification. The applied model is the following:

$$\max \sum_{j=1}^{N} d_{it} \cdot y_{it} \tag{32}$$

subject to

$$\sum_{j \in F, tt_{jit} \le r} x_{jt} \cdot C + \sum_{j \in W, tt_{jit} \le r} 1C \ge y_{it} \cdot d_{it}, \forall i \in D, \forall t \in T \tag{33}$$

$$\sum_{j=1 \in J} x_{jt} \le 1, \forall t \in T \tag{34}$$

Where x_{jt} is a binary variable which is equal to 1 if there is a vehicle in location j at time t and 0 otherwise, instead of representing the number of vehicles as previously.
The variable d_{it} defines the weight of each node, according to the proportion of demand expected in that site for each of the time periods. Defining the requirement, r equal to seven minutes, there can be guaranteed, in theory, 97% of the demand, positioning the extra vehicle in the candidate location 3 between 12 a.m and 6 p.m. and another repositioning for the next period in candidate site number 6, from 6 p.m. to 12 p.m.. When r is reduced to five minutes, the optimal solution is to position the vehicle in the candidate position 6 for the period number 3, providing a coverage of 87%.
As the candidates were not specified by the the CBSB and the choice was made based on the map of coverage presented previously, we applied a more complex model that considers that all demand node are also candidate location to position emergency vehicles during the four different periods of the day. Evidently there are costs and restrictions directly related to the change of position of this vehicles as for example: the cost associated with the travel itself and the difficulty to find a suitable parking lot that allows easy and fast exit of the vehicle in case of emergency. Also, there is little information available that allows to determine which specific sites are appropriate to park the vehicles. This way, instead of using all the single nodes as independent candidates, these were organized in a grid of approximately two km per two km. Hence, there is a higher probability that the approximation of coverage is correct and the final site can be chosen within the limits of the quadrant. To do so, some approximations needed to be done. First, the expected demand of each quadrant is the sum of the expected demand of all the nodes inside its limits. Secondly, to estimate the travel times, we followed a pessimist approach which means that the estimated travel time from each quadrant to all the demand nodes is the maximum travel time that any of the nodes inside the quadrant takes to each one of the demand nodes, i.e., the maximum time possible from the group of nodes contained in each grid unit. The linear programming model used to solve this specific problem is presented at continuity:

$$\max \sum_{i=1}^{N} \sum_{t=1}^{T} d_{it} \cdot y_{it} - \sum_{j=1 \in J} \sum_{k=1 \in J} \sum_{t=1}^{T} u_{jkt} - \sum_{j=1 \in F} \sum_{t=1}^{T} x_{jt} \tag{35}$$

subject to

$$\sum_{j \in J, tt_{jit} \le 7} x_{jt} \ge y_{it}, \forall i \in D, \forall t \in T \tag{36}$$

$$\sum_{i=1}^{N} d_{it} \le \sum_{j \in F, tt_{jit} \le 7} x_{jt} \cdot c, \forall t \in T \tag{37}$$

$$\sum_{j \in J} x_{jt} \leq M, \forall t \in T \tag{38}$$

$$x_{jt} + \sum_{k \in J} u_{kj(t+1)} - \sum_{k \in J} u_{jk(t+1)} = x_{j(t+1)}, \forall t \in T, \forall j \in J \tag{39}$$

$$x_{jT} + \sum_{k \in J} u_{kj1} - \sum_{k \in J} u_{jk1} = x_{j1}, \forall t \in T, \forall j \in J \tag{40}$$

$$y_{it} \in \{0,1\} \forall t \in T, \forall i \in D \tag{41}$$

$$x_{jt} \in \{0,1,...,M\} \subset \mathbb{N}_0 \forall t \in T, \forall j \in J \tag{42}$$

$$u_{jkt} \in \{0,1,...,M\} \subset \mathbb{N}_0 \forall t \in T, \forall j \in J, \forall k \in J \tag{43}$$

The solution of the optimization model was determined using Pyomo - glpk solver to optimally and is shown in the next table.

Table 5: Results of the optimization model with flexible locations and maximal coverage

	t=1	t=2	t=3	t=4
Fire station 1 and 2	5	5	4	4
Fire station 3	2	2	2	2
Fire station 4	3	3	3	3
Fire station 5	4	4	4	4
Fire station 6	2	2	2	2
Fire station 7	2	2	2	2
Fire station 8	2	2	2	2
Flexible	0	0	1	1
Number of vehicles $\sum_{j=1 \in F} x_{jt}$	20	20	20	20
Number of relocations $\sum_{j=1}^{M} \sum_{k=1}^{M} u_{jkt}$	1	0	1	0

The optimal solution obtained by solving this model guaranties a coverage of 89% by placing a vehicle in the region marked in the following figure, between 12 a.m. and 12 p.m., while keeping the rest of the vehicles in their usual fireman station. It is observable that the flexible locations are merely used in high frequency periods (t = 3, t = 4). According to the expected coverage, this solution is similar to the one obtain with the previous and simpler model. That said, it would be of interest to better estimate and collect information about the cost having a vehicle outside the assigned facility.

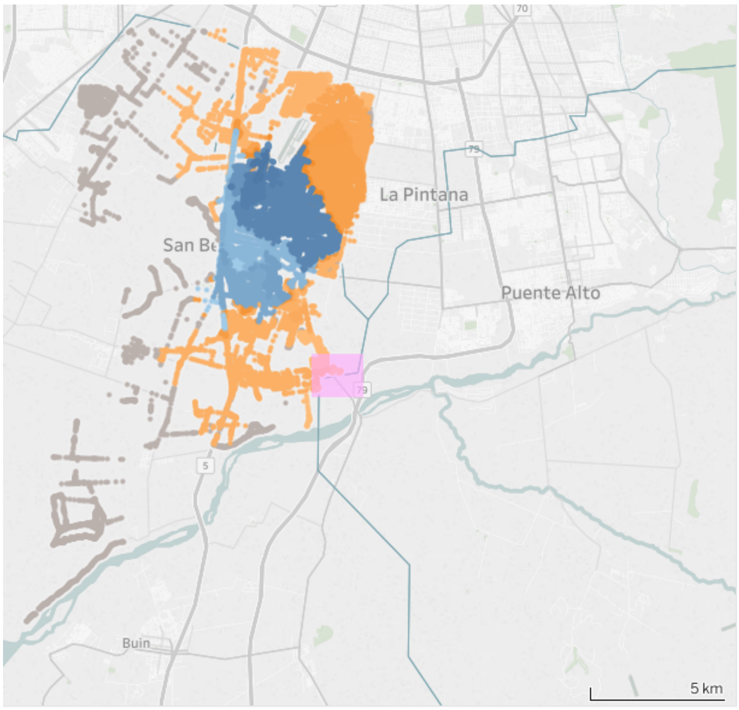

Figure 14: Demand nodes and respective area not covered of the municipality of San Bernardo with solution implementation

As exposed in the preliminary analysis of the historical data, the eight existing fire stations of CBSB were able to cover 63% of the emergencies during 2016 and 2017 (in seven minutes). Considering the most conservative, the proposed solution (placing a vehicle between 12 a.m. and 12 p.m. in the southern limit of the municipality) would increase 26% the coverage of the expected demand. In other words, the application of this configuration guarantees that every day 2 more emergencies are attended in seven minutes which according to the fire department of Santiago can reflect in a major decrease in the number of severe victims.To better understand the improvement in terms of time savings, we calculated the minimum time to arrive to that region using the original configuration. In average, it will take 3,2 minutes less to arrive to emergencies inside the coverage of the quadrant in the previous figure.
Although these results seem promising, one must note that the presented coverage estimative does not take into account the vehicle availability.
To compare the proposed solution with the present configuration, in terms of geographical coverage, we calculated the proportion of nodes that have a vehicle within a seven minute radius, for the different periods of the day. This results are presented in the next table. It is important to note that do not take into account the expected demand at each site (as used in the objective function), neither the capacity of the vehicles, i.e., the number of emergencies allocated to each station or vehicle can surpass the respective capacity and the node would considered covered.

Table 6: Nodes coverage comparison between solutions

Solution	t=1	t=2	t=3	t=4
Actual coverage	89%	75%	69%	73%
Configuration with one flexible location	89%	75%	80%	85%

As the vehicle availability in the different facilities is not known, it is possible that the capability for each fire station is overestimated. Particularly, a station to which are theoretically assigned three fire fighting vehicles can be operating for days with only one if there is not enough personal available to operate the vehicles. This way, the real capacity should be studied in order to determine minimum number of operational vehicles needed in each fireman station to guarantee coverage.

4.1.4 Post Optimality Analysis

Even thought, the specific problem to solve was the positioning of a single vehicle in one of the determined candidate locations, the results raise other questions. To start with, the number of vehicles outside the fire stations are needed to guarantee total coverage for all periods of the day, i.e., estimated coverage of 100%.

Using the previous mathematical formulation (35)-(43), we concluded that it it needed to have five flexible locations for all the four established periods of the day. This solution is equivalent to adding five permanent locations to the initial configuration in the candidate locations: 1, 3, 4, 6 and 8.

With this result, it was also concluded that it is pertinent not only to classify the emergencies according to equipment needs but also, define the events that need to be attended in seven minutes, as fire man do not only provide medical support but also, animal rescue or non-urgent rescues.

Also, we studied the sensitivity of the solution to our artificially estimated values such as the vehicle capacity. For the respective restriction, i.e. capacity restriction (37), to limit the optimal solution, the fire fighting vehicles had to have an availability of 12% of the time. Although this number may seem too small compared to expected, it is important to mention that all the fire units, in Santiago de Chile, are composed by voluntary fire man. Thus, not all stations are available at all times. At the time of this research there was no information available about the availability and capacity of each fire station and so this estimation cannot be properly evaluated.

To support managing this particular issue, we developed a display which allows the CBSB to analyse using a map the different regions of the commune which were not covered when a range of selected station is operational and based on our travel time estimations.

In the figure 15, there is the interface used to provide that information online.

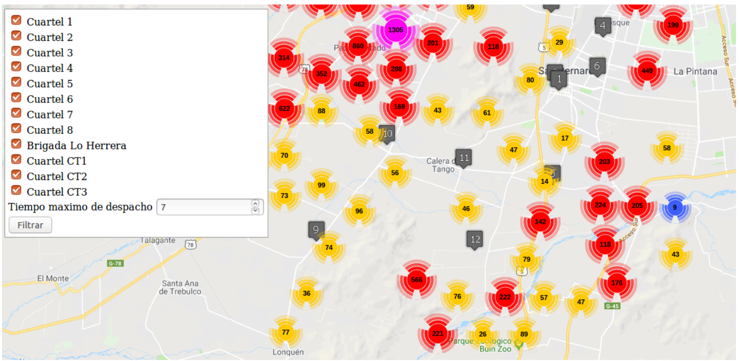

Figure 15: Not covered nodes in the commune of San Bernardo, El Bosque and Calera de Tango

4.2 Calera de Tango

During this research, the CBSB faced a new challenge: to determine the best permanent location for an emergency vehicle in the municipality of *Calera de Tango*. There was no historical data of demand but the firemen indicated four candidate locations. For this, a simpler model was developed.

$$\max \sum_{j=1}^{N} d_i \cdot y_i \tag{44}$$

subject to

$$\sum_{j \in J, tt_{ji} \leq 7} x_j \geq y_i, \forall i \in D \tag{45}$$

$$\sum_{j=1}^{M} x_j \leq 1 \tag{46}$$

Following the nomenclature of the previous model,d_i is the vector of percentage of expected demand at site i, i.e. $\sum_{i=1}^{N} d_i = 1$. As the lack of data did not support the construction of a proper forecasting model, these values were obtained with a random distribution. The variable decisions are y_i and x_j. The first is a binary variable that is equal to 1 if the node i is covered within a radius of seven minutes by the vehicle. The second is also a binary variable that is equal to one if the vehicle should be position in the candidate location j. The objective function (1) maximizes the coverage, while equation (2) ensures that only the demand nodes located 7 minutes away from the vehicle site are considered covered. Constraint (3) limits the number of vehicle locations.
To solve the problem, we took into account 405 demand nodes distributed in the entire municipality as shown in figure 16 and generated 300 random distributions to attribute a percentage of demand to each of those sites. The travel time table, in this specific application, does not depend on time as the solution that we were looking for can not include relocations during the day. Instead, we used the travel time period for the worst case scenario, i.e., hour 7:30 which has the biggest travel times. The solution of the optimization model was obtained using *GNU Linear Programming Kit* from *Pyomo* solver, copyright 2017 National Technology and Engineering Solutions of Sandia and provides the vehicle location marked at the image of the municipality nodes which guarantees a coverage of around 99,25%,in average. The locations that are not covered with the presented solutions are also marked in the figure.

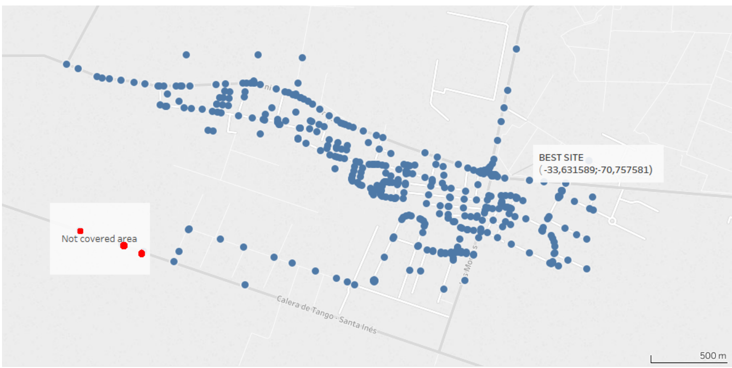

Figure 16: Demand nodes, vehicle location and respective area not covered of the municipality of Calera de Tango

The nodes are a simplification that aims to represent the entire area of the municipality. As exposed previously the emergencies are more common to occur where the population concentration is higher. This way, the sites represented in the image above are mainly concentrated in the residential area of the municipality that is delimited in the following image with a ride line. The region that is not covered is represented with three nodes previously corresponds to rural parcels as can be seen in figure 17.

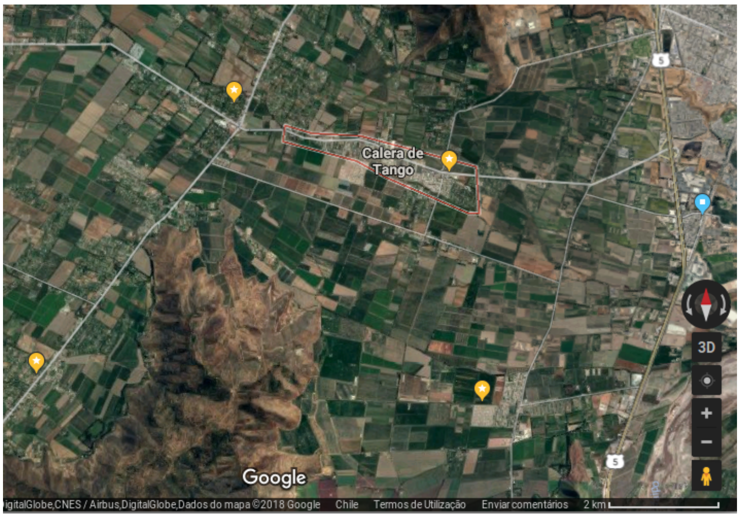

Figure 17: Satellite view of the municipality of Calera de Tango using Google Maps - july 5 2018^{th}

In the previous figure, the four candidate locations are represented by yellow balloons with white stars and the closest firemen station of CBSB by a blue balloon with a white square. Given the results obtained, we proceed to reduce the required response time to five minutes and still got a high coverage of around 87,19%. That said we enlarged the area to cover by adding the nodes represented in the following figure, instead of focusing on the residential zone of the municipality, representing an area of approximately 50 km ^{2}.

Figure 18: Demand nodes, vehicle location and respective covered area of the municipality of Calera de Tango

To solve this problem it was also taken into account as available resources the fireman stations of San Bernardo, but these cannot arrive to any of the emergency sites within the seven minutes response time required. The best localization is the same obtain previously but only guaranties a coverage of around 35,12%. These nodes are the green points in the map. This means that to cover all *Calera de Tango*, at the worst case scenario, i.e. speed profiles of 7:30 a.m., there should be added at least one more vehicle deployment site. To do so, constraint (3) was changed to permit a total number of vehicles from 0 to 2. Following the same solution strategy we obtained two different solutions that combine the best site of the previous solutions with two other candidate sites. The candidate sites used in this solution and the respective covered area are represented in figure 19. 4 of the 300 tested distribution pointed a solution with the vehicles 1 and 3 that guaranties an average coverage of 50,12%. For the remaining 296 distribution the most two appropriate site for the vehicles are the position 1 and 2 that cover 51,13% of the demand, in average.

Figure 19: Covered nodes by each vehicle: green for vehicle 1, blue for vehicle 2, red for vehicle 3 and yellow for the nodes with double coverage for the configuration that includes the vehicles 1 and 2

5 Conclusions and Future Work

Analysis of extensive real data indicate that significant variations in time occur not only in travel time between different points of the city but also in the demand pattern. This way, using equal travel time during rush hours and overnight oversimplifies reality often leading to overestimation of the covering and underestimating of the required resources. In the same way, fixing a single probability based only on the geographical distribution of the events is too simplistic and does not accurately represent reality.

On this dissertation an optimization model that takes into account the these variations for fire fighting vehicles locating and relocating is presented. The discussed model allows to relocate a specified number of vehicles to flexible locations in order to maximize the expected coverage, i.e., we propose the use of strategic locations outside fire stations for fire fighting vehicles during specific periods of the day.

Our model differs from previous related work in crucial aspects such as the travel time being time-dependent. With the simultaneous consideration of various dynamic and flexible vehicle locations, the resources can be temporarily relocated and used efficiently. As real life organizations have to manage human and capital resources, the degree of flexibility is limited and varies with operational specificities, we use penalty values in the objective function for relocations and street vehicle locations, which allow the decision maker to directly influence the weight of these factors on solving the location problem.

As highlighted by Degel et al (2015), the fixed coverage does not explicitly consider the number of parallel call and consequently the real number of needed vehicles, i.e., if there are more than two parallel operations, this objective does not ensure enough available vehicles. The studied model minimizes this error by dividing the day into four periods but still it cannot ensure the non-existence of simultaneous emergencies. This way, we suggest an investigation on the probability of parallel calls in order to define an empirical time-dependent coverage or, in alternative, the minimum number of divisions of the day in periods of time that avoids this estimation errors.

During this dissertation, little literature was found related with the prediction of emergencies geographical and chronologically, simultaneously. The quality of such prediction is crucial to increase the accuracy of the estimation of fleet capacity and resource location and allocation. This way, the forecasting techniques applied specifically to occurrence of emergency should be developed.

Although there is much literature on location problems, there are still developments to be made. For instance, the use of probability distributions of the arrival time instead of average values. It is of particular importance firstly to define the fire station to add a factor of certainty to the estimated travel times. This is already being studied inside this project and when applied to the dispatching system and, consequently, to the location problem will increase the quality of the solutions.

To conclude, this study has achieved the proposed goals of the project FONDEF R.A.P.I.D.O.. The proposed solution is predicted to improve the emergency coverage from around 63% (real) to 97%. The location problem and the achieved results were also presented to the fire department of *Santiago de Chile*. This was particularly relevant since the presented methodology can be adapted and applied to the other municipalities of *Santiago de Chile*.

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Appendix A Types of emergencies

Codes for the different type of emergencies	
10-0	Structural Call
10-1	Vehicle Call
10-2	Grassland and/or Trash Call
10-3	Emergency Rescue Call
10-4	Vehicle Rescue Call
10-5	Dangerous Substance Call
10-6	Gas Emanation Call
10-7	Electric Problem Call
10-8	No Classified Call
10-9	Other Services
10-10	Rubble Call
10-11	Support to Airports and/or Airfield
10-12	Support to other Fire Departments
10-13	Terrorist Attack Cal
10-14	Aerial Accident Call
10-15	Simulation Call
10-16	Emergency in Tunnel

Appendix B Types of Vehicles

Code Type	Characteristics
B, BR, BX	vehicles equipped with a motor to bomb water and the can transport water to the emergency site
H	vehicles specialized to transport dangerous substances
J, S, X	vehicles with specialized equipment
K, M	command vehicles
LT	B4, J4, RX4, BX4, RH4
Q	vehicles equiped with flexible stairs for rescuing
R,RH,RX	rescuing vehicles
Z	vehicles with large water deposits used to transport water to the emergencies

Appendix C Fire Station and Vehicle Allocation

Fire Station	Vehicles
1	B1, S1, X1, K1, LT1
2	K2, Q2, R2
3	B3, H3, K3, X3
4	B4, J4, RX4, BX4, RH4
5	B5, H5, K5, S5
6	B6, BR6, BX6
7	B7, BR7, BX7
8	B8, BX8

Appendix D Variables and Parameters of Theoretical Model

Variable/Parameter	Description
z_l	objective value
λ_l	weight of objective l in the objective function
d^1_{it}, d^2_{it}	expected demand at node i during period t for each type of emergency 1 and 2
y^1_{it}, y^2_{it}	1 if node i is covered during period t, 0 otherwise, for each type of emergency 1 and 2
u^A_{jkt}, u^B_{jkt}	number of relocations from node j to node k before the period t for each vehicle type A and B
x_{jt}	number of vehicles located in the node j during period t
c^A, c^B	capacity of vehicle of type A and B respectively
M^A, M^B	number of vehicles available of each type